

Quantum correlation, monogamy relation, and quantum phase transitions in two-dimensional XY spin system

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Abstract

The purpose of the paper is mainly to investigate the quantum critical behavior of two-dimensional XY spin system by calculating the quantum correlation and monogamy relation through implementation of quantum renormalization group theory. Numerical analysis indicates that quantum correlation as well as quantum nonlocality can be used to efficiently detect the quantum critical property in two-dimensional XY spin system. The nonanalytic behavior of the first derivative of the quantum correlation approaches infinity and the critical point is reached as the size of the model increases. Furthermore, we discuss the quantum correlation distribution in this model based on the square of concurrence (SC) and square of quantum discord (SQD). The monogamous properties of SC and SQD are obtained for this model. Particularly, we show that the monogamy score can be used to capture the quantum critical point.

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I. INTRODUCTION

In order to show the nonlocality in quantum mechanics, Einstein, Podolsky, and Rosen proposed the thought experiment known as the EPR paradox in 1935 [1]. Such kind of nonlocality is defined as entanglement later. The investigation on the nonlocality of quantum physics has formed a new discipline namely quantum information science. It is generally recognized that entanglement is the key resource in quantum information science [2]. In recent years, the study on the monogamy properties of entangled state has attracted much attention, i.e., quantum entanglement cannot be freely shared among the constituents of a multipartite system [3, 4]. Monogamy is one of the basic rules in making quantum cryptography secure and plays an indispensable role in superdense coding [5]. Many achievements have been obtained after introduced by Coffman et al. However, according to recent progress, there are also some states containing quantum correlation beyond entanglement and are also very effective in quantum information processing [6]. So, entanglement can not signify all the quantum nonlocality in a quantum system. Quantum correlation may be the most fundamental resource in quantum information protocols. Therefore, monogamy relation of quantum correlation also gets much attention [7–9]. Motivated by the development of monogamy relation of quantum correlation, we also want to ask whether the monogamy relation can be used to investigate some fundamental physics problems, such as quantum phase transitions.

Quantum phase transitions (QPT) [10–13] indicates that the ground state of a many-body system changes abruptly when varying a physical parametersuch as magnetic field or pressure at absolute zero temperature. Contrary to thermal phase transitions, QPT is completely induced by quantum fluctuations. Generally, researchers adopt order parameter, correlation functions and other concepts in thermal phase transition to investigate QPT. Though many meaningful results have been got, there is still some shortage in it. The rapid development of quantum information science provides us a good means to understand the nature of QPT. A lot of studies [14–17] indicate that the concept of entanglement and quantum correlation can be used to detect QPT or describe the property near the critical point. In addition, the renormalization group theory has been adopted as one important measure to study QPT for many years. Recently, researchers have begun to study the QPT in low-dimensional spin systems by combing quantum information concepts and quantum renormalization group

(QRG) theory. It has been shown that the behavior of the entanglement in the vicinity of the critical point is directly connected to the quantum critical properties [18–22]. The quantum correlation measures also can be used to detect the quantum critical points associated with quantum phase transitions after implementation of the QRG method [23–25]. However, these study mainly concentrate on the one-dimensional systems. The two-dimensional spin systems, such as lanar quadrilateral spin crystal, triangular spin grid and kagome spin lattice also are very important low-dimensional systems. The research on these models will promote the understanding of ground-state properties, correlation length, and critical point in two-dimensional system.

Usman [26] has given an analysis of such model by using entanglement and QRG. They conjectured that the system reaches the critical point in the lesser number of QRG iterations as compared with the one-dimensional model. Nevertheless, as mentioned before, quantum entanglement is not adequate to represent all the quantum correlation contained in a quantum system, so this inspires us to apply the quantum correlation measures to study such two-dimensional model [23]. Furthermore, another question concerning monogamy relation also deserves our attention. Whether the monogamy relation exists in two-dimensional spin system? Whether the monogamy relation can be used to catch the quantum critical point? If so, can we find some useful tool to demonstrate it? To answer these questions, the dynamical behavior and the monogamy property of the two-dimensional XY system will be studied by the quantum correlation measures.

In the next section, we will give an introduction on the two-dimensional XY model. In Sec. III, the quantum correlation measures and the analytical results of this model are given. In Sec. IV, the dynamical behavior, nonanalytic results, and scaling behavior are presented. In Sec. V, the monogamy relation of this system is given. The last section is a summary of our work.

II. MODEL DESCRIPTION

The Hamiltonian of a two-dimensional XY model can be written as

$$H(J, \gamma) = \frac{J}{4} \sum_{i=1}^N \sum_{j=1}^N [(1 + \gamma)(\sigma_{i,j}^x \sigma_{i+1,j}^x + \sigma_{i,j}^x \sigma_{i,j+1}^x) + (1 - \gamma)(\sigma_{i,j}^y \sigma_{i+1,j}^y + \sigma_{i,j}^y \sigma_{i,j+1}^y)] \quad (1)$$

where J is the exchange interaction, γ is the anisotropy parameter and $\sigma^\tau(\tau = x, y)$ are standard Pauli operators at site i, j . In order to apply QRG, we need to select five-site as one block. Such five-site blocks can be viewed as one-site in renormalized subspace. A schematic diagram can be seen in Fig. 1.

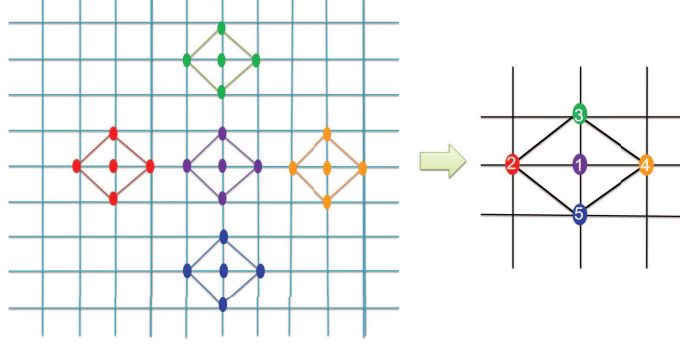


FIG. 1: (Color online) A schematic description of QRG for five sites in a block.

So, the Hamiltonian can be separated as block Hamiltonian H^B and interacting Hamiltonian H^{BB} respectively.

$$H^B = \frac{J}{4} \sum_{L=1}^{N/5} [(1 + \gamma)(\sigma_{L,1}^x \sigma_{L,2}^x + \sigma_{L,1}^x \sigma_{L,3}^x + \sigma_{L,1}^x \sigma_{L,4}^x + \sigma_{L,1}^x \sigma_{L,5}^x) + (1 - \gamma)(\sigma_{L,1}^y \sigma_{L,2}^y + \sigma_{L,1}^y \sigma_{L,3}^y + \sigma_{L,1}^y \sigma_{L,4}^y + \sigma_{L,1}^y \sigma_{L,5}^y)] \quad (2)$$

$$H^{BB} = \frac{J}{4} \sum_{L=1}^{N/5} [(1 + \gamma)(\sigma_{L,2}^x \sigma_{L+1,3}^x + \sigma_{L,2}^x \sigma_{L+1,4}^x + \sigma_{L,2}^x \sigma_{L+2,5}^x + \sigma_{L,3}^x \sigma_{L+2,4}^x + \sigma_{L,3}^x \sigma_{L+2,5}^x + \sigma_{L,4}^x \sigma_{L+3,5}^x) + (1 - \gamma)(\sigma_{L,2}^y \sigma_{L+1,3}^y + \sigma_{L,2}^y \sigma_{L+1,4}^y + \sigma_{L,2}^y \sigma_{L+2,5}^y + \sigma_{L,3}^y \sigma_{L+2,4}^y + \sigma_{L,3}^y \sigma_{L+2,5}^y + \sigma_{L,4}^y \sigma_{L+3,5}^y)] \quad (3)$$

The two lowest eigenvectors of the corresponding L -th block

$$\begin{aligned} |\varphi_0\rangle = & \gamma_1(|\uparrow\uparrow\uparrow\uparrow\downarrow\rangle + |\uparrow\uparrow\uparrow\downarrow\uparrow\rangle + |\uparrow\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\uparrow\uparrow\rangle) + \\ & \gamma_2(|\uparrow\uparrow\downarrow\downarrow\downarrow\rangle + |\uparrow\downarrow\uparrow\downarrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\downarrow\uparrow\rangle) + \\ & \gamma_3(|\downarrow\uparrow\uparrow\uparrow\uparrow\rangle + \gamma_4(|\downarrow\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\downarrow\uparrow\rangle + \\ & |\downarrow\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\downarrow\uparrow\rangle + |\downarrow\downarrow\downarrow\uparrow\uparrow\rangle) + \gamma_5|\downarrow\downarrow\downarrow\downarrow\downarrow\rangle) \end{aligned} \quad (4)$$

$$\begin{aligned}
|\varphi'_0\rangle = & \gamma_6 | \uparrow\uparrow\uparrow\uparrow\uparrow \rangle + \gamma_7 (| \uparrow\uparrow\uparrow\downarrow\downarrow \rangle + | \uparrow\uparrow\downarrow\uparrow\downarrow \rangle + | \uparrow\uparrow\downarrow\downarrow\uparrow \rangle \\
& + | \uparrow\downarrow\uparrow\uparrow\downarrow \rangle + | \uparrow\downarrow\uparrow\downarrow\uparrow \rangle + | \uparrow\downarrow\downarrow\uparrow\uparrow \rangle) + \gamma_8 | \uparrow\downarrow\downarrow\downarrow \rangle \\
& + \gamma_9 (| \downarrow\uparrow\uparrow\uparrow\downarrow \rangle + | \downarrow\uparrow\uparrow\downarrow\uparrow \rangle + | \downarrow\uparrow\downarrow\uparrow\uparrow \rangle + | \downarrow\downarrow\uparrow\uparrow\uparrow \rangle) \\
& + \gamma_{10} (| \downarrow\uparrow\downarrow\downarrow\downarrow \rangle + | \downarrow\downarrow\uparrow\downarrow\downarrow \rangle + | \downarrow\downarrow\downarrow\uparrow\downarrow \rangle + | \downarrow\downarrow\downarrow\downarrow\uparrow \rangle)
\end{aligned} \tag{5}$$

can be used to establish the projection operator $P_0^L = |\varphi_0\rangle_L \langle \uparrow | + |\varphi'_0\rangle_L \langle \downarrow |$. Analytical expressions for the γ_i s can be found in the Ref.[26], $\langle \uparrow |$, $\langle \downarrow |$ are renamed states of each block to represent the effective site degrees of freedom. The effective Hamiltonian can be defined as

$$\begin{aligned}
H^{eff}(J', \gamma') &= P_0^\dagger (H^B + H^{BB}) P_0 \\
&= \frac{J'}{4} \sum_{p=1}^{N/5} \sum_{q=1}^{N/5} [(1 + \gamma')(\sigma_{p,q}^x \sigma_{p+1,q}^x + \sigma_{p,q}^x \sigma_{p,q+1}^x) + (1 - \gamma')(\sigma_{p,q}^y \sigma_{p+1,q}^y + \sigma_{p,q}^y \sigma_{p,q+1}^y)]
\end{aligned} \tag{6}$$

where the renormalized couplings are

$$\begin{aligned}
J' = & J(\gamma_{10}^2(9\gamma_4^2 + 6\gamma\gamma_4\gamma_5 + \gamma_5^2) + 9\gamma_2^2\gamma_7^2 + \gamma_1^2(\gamma_6^2 + 6\gamma\gamma_6\gamma_7 + 9\gamma_7^2) + 6\gamma\gamma_2^2\gamma_7\gamma_8 + \gamma_2^2\gamma_8^2 + 6\gamma\gamma_2\gamma_3\gamma_7\gamma_9 \\
& + 18\gamma\gamma_2\gamma_4\gamma_7\gamma_9 + 2\gamma_2\gamma_3\gamma_8\gamma_9 + 6\gamma\gamma_2\gamma_4\gamma_8\gamma_9 + \gamma_3^2\gamma_9^2 + 6\gamma\gamma_3\gamma_4\gamma_9^2 + 9\gamma_4^2\gamma_9^2 + 2\gamma_1\{\gamma_2[3\gamma_7(3\gamma\gamma_7 + \gamma_8) \\
& + \gamma_6(3\gamma_7 + \gamma_8)] + (\gamma\gamma_3\gamma_6 + 3\gamma_4\gamma_6 + 3\gamma_3\gamma_7 + 9\gamma\gamma_4\gamma_7)\gamma_9\} + 2\gamma_{10}\{\gamma_1(\gamma_5\gamma_6 + 9\gamma_4\gamma_7) + \gamma[9\gamma_2\gamma_4\gamma_7 \\
& + 3\gamma_1(\gamma_4\gamma_6 + \gamma_5\gamma_7) + \gamma_2\gamma_5\gamma_8 + 9\gamma_4^2\gamma_9 + \gamma_3\gamma_5\gamma_9] + 3[\gamma_2(\gamma_5\gamma_7 + \gamma_4\gamma_8) + \gamma_4(\gamma_3 + \gamma_5)\gamma_9]\})
\end{aligned} \tag{7}$$

$$\begin{aligned}
\gamma' = & [2(3\gamma_{10}\gamma_4 + 3\gamma_1\gamma_7 + \gamma_2\gamma_8 + \gamma_3\gamma_9)(\gamma_{10}\gamma_5 + \gamma_1\gamma_6 + 3\gamma_2\gamma_7 + 3\gamma_4\gamma_9) + \gamma(\gamma_{10}^2(9\gamma_4^2 + \gamma_5^2) + 9\gamma_2^2\gamma_7^2 \\
& + \gamma_1^2(\gamma_6^2 + 9\gamma_7^2) + \gamma_2^2\gamma_8^2 + 18\gamma_2\gamma_4\gamma_7\gamma_9 + 2\gamma_2\gamma_3\gamma_8\gamma_9 + \gamma_3^2\gamma_9^2 + 6\gamma_1[\gamma_2\gamma_7(\gamma_6 + \gamma_8) + (\gamma_4\gamma_6 + \gamma_3\gamma_7)\gamma_9] \\
& + 2\gamma_{10}\{\gamma_1(\gamma_5\gamma_6 + 9\gamma_4\gamma_7) + 3[\gamma_2(\gamma_5\gamma_7 + \gamma_4\gamma_8) + \gamma_4(\gamma_3 + \gamma_5)\gamma_9]\})]/[\gamma_{10}^2(9\gamma_4^2 + 6\gamma\gamma_4\gamma_5 + \gamma_5^2) + 9\gamma_2^2\gamma_7^2 \\
& + \gamma_1^2(\gamma_6^2 + 6\gamma\gamma_6\gamma_7 + 9\gamma_7^2) + 6\gamma\gamma_2^2\gamma_7\gamma_8 + \gamma_2^2\gamma_8^2 + 6\gamma\gamma_2\gamma_3\gamma_7\gamma_9 + 18\gamma_2\gamma_4\gamma_7\gamma_9 + 2\gamma_2\gamma_3\gamma_8\gamma_9 + 6\gamma\gamma_2\gamma_4\gamma_8\gamma_9 \\
& + \gamma_3^2\gamma_9^2 + 6\gamma\gamma_3\gamma_4\gamma_9^2 + 9\gamma_4^2\gamma_9^2 + 2\gamma_1\{\gamma_2[3\gamma_7(3\gamma\gamma_7 + \gamma_8) + \gamma_6(3\gamma_7 + \gamma_8)] + (\gamma\gamma_3\gamma_6 + 3\gamma_4\gamma_6 \\
& + 3\gamma_3\gamma_7 + 9\gamma\gamma_4\gamma_7)\gamma_9\} + 2\gamma_{10}(\gamma_1(\gamma_5\gamma_6 + 9\gamma_4\gamma_7) + \gamma[9\gamma_2\gamma_4\gamma_7 + 3\gamma_1(\gamma_4\gamma_6 + \gamma_5\gamma_7) + \gamma_2\gamma_5\gamma_8 \\
& + 9\gamma_4^2\gamma_9 + \gamma_3\gamma_5\gamma_9] + 3[\gamma_2(\gamma_5\gamma_7 + \gamma_4\gamma_8) + \gamma_4(\gamma_3 + \gamma_5)\gamma_9])]
\end{aligned} \tag{8}$$

The ground state density matrix is given by

$$\rho = |\varphi_0\rangle\langle\varphi_0| \quad (9)$$

Owing to the symmetry, the bipartite state between the center block and every corners block is identical that is $\rho_{12} = \rho_{13} = \rho_{14} = \rho_{15}$. Similarly, $\rho_{23} = \rho_{34} = \rho_{45} = \rho_{25}$. After some algebra, we can derive the bipartite state ρ_{12} and ρ_{23} by tracing out particles 3,4,5 or 1,4,5.

$$\rho_{12} = \begin{pmatrix} 3\gamma_1^2 + \gamma_2^2 & 0 & 0 & 3\gamma_1\gamma_4 + \gamma_2\gamma_5 \\ 0 & \gamma_1^2 + 3\gamma_2^2 & \gamma_1\gamma_3 + 3\gamma_2\gamma_4 & 0 \\ 0 & \gamma_1\gamma_3 + 3\gamma_2\gamma_4 & \gamma_3^2 + 3\gamma_4^2 & 0 \\ 3\gamma_1\gamma_4 + \gamma_2\gamma_5 & 0 & 0 & 3\gamma_4^2 + \gamma_5^2 \end{pmatrix} \quad (10)$$

$$\rho_{23} = \begin{pmatrix} 2\gamma_1^2 + \gamma_3^2 + \gamma_4^2 & 0 & 0 & 2\gamma_1\gamma_2 + \gamma_3\gamma_4 + \gamma_4\gamma_5 \\ 0 & \gamma_1^2 + \gamma_2^2 + 2\gamma_4^2 & \gamma_1^2 + \gamma_2^2 + 2\gamma_4^2 & 0 \\ 0 & \gamma_1^2 + \gamma_2^2 + 2\gamma_4^2 & \gamma_1^2 + \gamma_2^2 + 2\gamma_4^2 & 0 \\ 2\gamma_1\gamma_2 + \gamma_3\gamma_4 + \gamma_4\gamma_5 & 0 & 0 & 2\gamma_2^2 + \gamma_4^2 + \gamma_5^2 \end{pmatrix} \quad (11)$$

III. DIFFERENT QUANTUM CORRELATION MEASURES

Next, we use quantum correlation measures and quantum nonlocality measure as well as Bell violation to investigate the quantum critical properties of this model.

A. Negativity

Negativity (Ne) [27] is an easily computable entanglement measure, it was introduced for testing the violation degree of positive partial transpose criterion in entangled states. Ne has been proved to be monotone and convex under local operations and classical communication. For a bipartite system ρ_{AB} , the partial transpose of ρ_{AB} on A can be described as $(\rho_{AB}^{T_A})_{ij,kl} = (\rho)_{kj,il}$. So, for a given state ρ_{AB} , the Ne is defined as

$$Ne(\rho_{AB}) = \frac{\|\rho_{AB}^{T_A}\| - 1}{2} \quad (12)$$

where $\|\rho_{AB}^{T_A}\| = Tr\sqrt{\rho_{AB}^{T_A}\rho_{AB}^{T_A\dagger}}$ denotes the trace norm. For the bipartite $2 \otimes 2$ and $2 \otimes 3$ quantum systems, Ne is the necessary and sufficient inseparable condition.

The analytical results of Ne for states ρ_{12} and ρ_{13} are expressed as

$$Ne_{12} = (9\gamma_1^4 + 6\gamma_1^2\gamma_2^2 + 4\gamma_1^2\gamma_3^2 - 18\gamma_1^2\gamma_4^2 - 6\gamma_1^2\gamma_5^2 + 24\gamma_1\gamma_2\gamma_3\gamma_4 + \gamma_2^4 + 30\gamma_2^2\gamma_4^2 - 2\gamma_2^2\gamma_5^2 + 9\gamma_4^4 + 6\gamma_4^2\gamma_5^2 + \gamma_5^4)^{1/2}/2 - (\gamma_2^2 + 3\gamma_4^2 + \gamma_5^2 + 3\gamma_1^2)/2. \quad (13)$$

$$Ne_{23} = (8\gamma_1^4 + 4\gamma_1^2\gamma_3^2 + 16\gamma_1^2\gamma_4^2 - 4\gamma_1^2\gamma_5^2 + 8\gamma_2^4 - 4\gamma_2^2\gamma_3^2 + 16\gamma_2^2\gamma_4^2 + 4\gamma_2^2\gamma_5^2 + \gamma_3^4 - 2\gamma_3^2\gamma_5^2 + 16\gamma_4^4 + \gamma_5^4)^{1/2}/2 - (\gamma_1^2 + \gamma_2^2 + \gamma_3^2/2 + \gamma_4^2 + \gamma_5^2/2). \quad (14)$$

B. Quantum Discord

As we know, the measure Ne is based on entanglement-separability paradigm. But Quantum discord (QD) is proposed from the perspective of information-theoretic paradigm. It is defined by the following expression [28]

$$QD(\rho_{AB}) = I(\rho_{AB}) - CC(\rho_{AB}), \quad (15)$$

where $I(\rho_{AB})$ is the total correlation and measured by the quantum mutual information $I(\rho_{AB}) = \sum_{i=A,B} S(\rho_i) - S(\rho_{AB})$, while $CC(\rho_{AB}) = S(\rho_A) - \min_{\Pi_k^B} S(S_{A|B}\{\Pi_k^B\})$, in which Π_k^B is a positive operator-valued measure performed on the subsystem B . $S(\rho) = -Tr(\rho \log_2 \rho)$ is the von Neumann entropy, ρ_A and ρ_B denotes the reduced density matrix of state ρ_{AB} by tracing out A or B .

Since ρ_{12} and ρ_{13} are X-type states, it is easy to compute the quantum discord result [2, 29, 30]. Even so, the analytical result for this case is too complicated to express it here. We mainly show the numerical result of QD in section IV.

C. Measurement-induced Disturbance

Measurement-induced disturbance (MID) [31] was defined by the difference between the two quantum mutual information of a quantum state ρ_{AB} and the corresponding post-measurement classical state $\Pi(\rho_{AB})$

$$MID(\rho_{AB}) = I(\rho_{AB}) - I(\Pi(\rho_{AB})), \quad (16)$$

here $I(\rho_{AB})$ is the same as in Eq. (15). $I(\Pi(\rho_{AB})) = \sum_{i,j} (\Pi_i^A \otimes \Pi_i^B) \rho_{AB} (\Pi_i^A \otimes \Pi_i^B)$ measures the classical correlation of a given state ρ_{AB} .

The analytical result of the *MID* for ρ_{12} and ρ_{13} are also very complicated and we shall not write it here. People can derive the eigenvalues and the diagonal element of ρ_{12} and ρ_{13} , and then deduced the results of *MID* for above two states.

D. Measurement-induced nonlocality

The measurement-induced nonlocality bases on the trace norm for a bipartite state ρ_{AB} expressed as [32]

$$MIN(\rho_{AB}) = \max_{\Pi^A} \|\rho_{AB} - \Pi^A(\rho_{AB})\|_1, \quad (17)$$

here $\|R\|_1 = Tr\sqrt{R^\dagger R}$, and the maximum is taken over the full set of local projective measurements Π^A that $\Pi^A(\rho_A) = \rho_A$.

The analytical result of *MIN* for ρ_{12} and ρ_{13} are expressed as

$$MIN_{12} = \max(|t_1|, |t_2|, |t_3|) \quad (18)$$

$$MIN_{23} = (|l_1 - l_2| + |l_1 + l_2|)/2 \quad (19)$$

where $t_1 = 2\gamma_1\gamma_3 + 6\gamma_4(\gamma_1 + \gamma_2) + 2\gamma_2\gamma_5$, $t_2 = 2\gamma_1\gamma_3 + 6\gamma_4(\gamma_2 - \gamma_1) - 2\gamma_2\gamma_5$, $t_3 = 2\gamma_1^2 - 2\gamma_2^2 - \gamma_3^2 + \gamma_5^2$, $l_1 = 2(\gamma_1 + \gamma_2)^2 + 2\gamma_4(\gamma_3 + \gamma_5) + 4\gamma_4^2$, $l_2 = 2(\gamma_1 - \gamma_2)^2 - 2\gamma_4(\gamma_3 + \gamma_5) + 4\gamma_4^2$, $l_3 = \gamma_3^2 - 2\gamma_4^2 + \gamma_5^2$.

E. Geometric Quantum Discord

The geometric measure of quantum discord is defined as [33]

$$GQD(\rho_{AB}) := \min_{\chi \in \Omega} \|\rho - \chi\|, \quad (20)$$

where Ω means the set of zero-discord states, whose general form is defined by $\chi = \sum_k p_k \Pi_k^A \otimes \rho_k^B$ with $0 \leq p_k \leq 1$ ($\sum_k p_k = 1$), and $\|\rho - \chi\|^2 = tr(\rho - \chi)^2$ means the square of the Hilbert-Schmidt norm.

The analytical results of the present model are expressed as

$$GQD_{12} = (t_1^2 + t_2^2 + t_3^2 + x_1^2 - \max(t_1^2, t_2^2, t_3^2 + x_1^2))/4 \quad (21)$$

$$GQD_{23} = (l_1^2 + l_2^2 + l_3^2 + x_2^2 - \max(l_1^2, l_2^2, l_3^2 + x_2^2))/4 \quad (22)$$

where $t_1, t_2, t_3, l_1, l_2, l_3$ are the same as Eq. (18) and Eq. (19), and $x_1 = 4\gamma_1^2 + 4\gamma_2^2 - \gamma_3^2 - 6\gamma_4^2 - \gamma_5^2$, $x_2 = 2\gamma_1^2 - 2\gamma_2^2 + \gamma_3^2 - \gamma_5^2$.

F. Bell Violation

The violation of Bell inequality is accepted as the existence of quantum nonlocality. Following equation is the Bell operator corresponding to the Clauser-Horne-Shimony-Holt (CHSH) form [34]

$$B = a \cdot \sigma \otimes (b + b') \cdot \sigma + a' \cdot \sigma \otimes (b - b') \cdot \sigma \quad (23)$$

where a, a', b, b' are the unit vectors in \mathbb{R}^3 , and the CHSH inequality can be written as $B = |\langle B_{CHSH} \rangle| = |\text{Tr}(\rho_{CHSH})| \leq 2$, in which the maximum violation of CHSH inequality is defined by

$$B_{CHSH}^{max} = \max_{a, a', b, b'} \text{Tr}(\rho B_{CHSH}) \quad (24)$$

The analytical result of B for ρ_{12} and ρ_{13} are expressed as

$$B_{12} = \sqrt{t_1^2 + t_2^2 + t_3^2 - \lambda_{min}} \quad (25)$$

$$B_{23} = \sqrt{l_1^2 + l_2^2 + l_3^2 - \kappa_{min}} \quad (26)$$

here $t_1, t_2, t_3, l_1, l_2, l_3$ also are the same as before, and $\lambda_{min} = \min(t_1^2, t_2^2, t_3^2)$, $\kappa_{min} = \min(l_1^2, l_2^2, l_3^2)$.

IV. RENORMALIZED QUANTUM CORRELATION

According to above-mentioned quantum correlation measures, the dynamical behavior of every quantity can be gotten by implementing QRG method.

A. Dynamics Behavior of Different Quantum Correlation

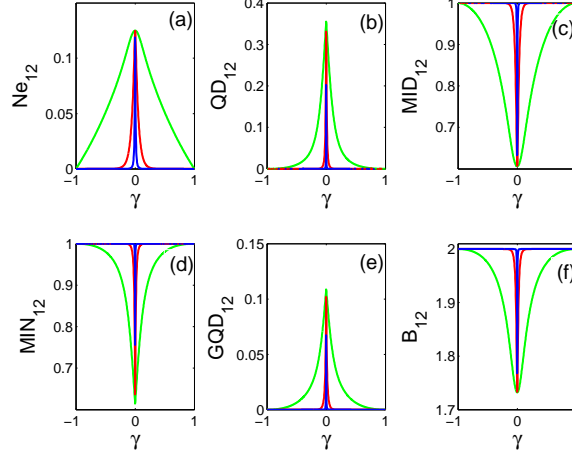


FIG. 2: (Color online) Different quantum correlation measures of ρ_{12} as a function of γ at different QRG steps. The green lines indicate 0-th step QRG, the red lines indicate 1-th step QRG, and the blue ones 2-th step QRG.

The properties of different quantum correlation measures versus γ in terms of QRG iterations are plotted in Fig. 2. The plots cross each other at the critical point $\gamma_c=0$. After two steps of renormalization, negativity will develop two saturated values, one that is nonzero for $\gamma_c=0$ and one that is zero for $\gamma_c \neq 0$. QD and GQD have the same property as entanglement. But the fixed value of MID and MIN are a little different, namely one that is nonzero for $\gamma_c=0$ and one that is 1 for $\gamma_c \neq 0$, the Bell inequality also have the same characteristic. Interestingly, we have found that the block-block correlations of ρ_{12} will demonstrate QPT at the critical point $\gamma_c=0$ and it cannot violate the CHSH inequality.

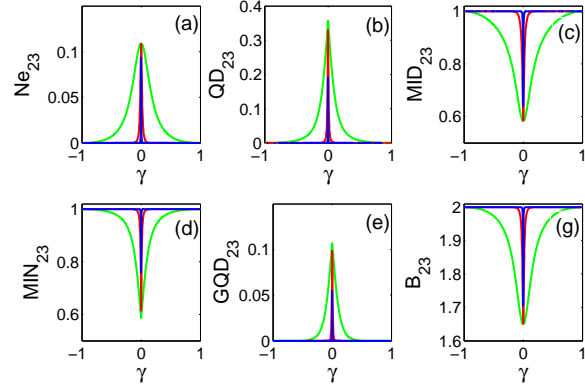


FIG. 3: (Color online) Different quantum correlation measures of ρ_{23} as a function of γ at different QRG steps. The plots color meaning are like before.

In Fig. 3, we illustrate the quantum correlation evolution of ρ_{23} versus γ for different QRG steps. The quantum correlation in ρ_{23} is between the two corner-site blocks when the system eventually become five-block state. The behavior of every quantum correlation measures are roughly the same with Fig. 2 but also have small difference. Such as the saturated value and the change rate vs γ .

B. Nonanalytic and scaling behavior

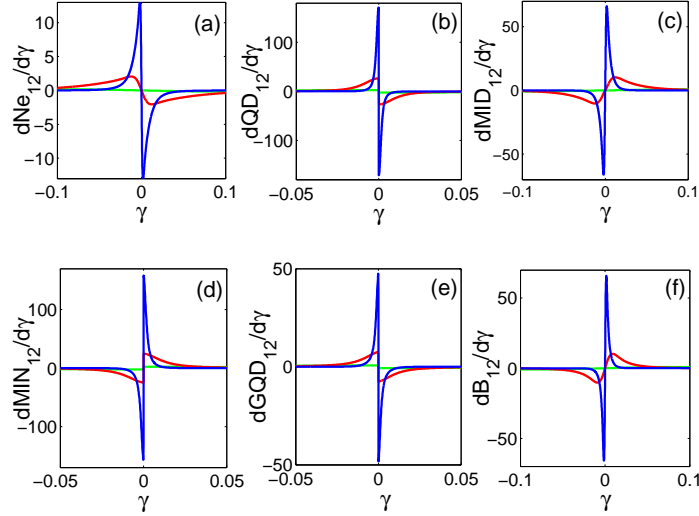


FIG. 4: (Color online) Evolution of the first derivative of quantum correlation measures under QRG for state ρ_{12} . The plots color meaning are like before.

We have shown the first derivative of different quantum correlation measures (DQCM) versus γ in Fig. 4. From this figure, we notice that the derivative of quantum correlation diverges at the critical point $\gamma = 0$ [20]. All the plots in the figure exhibit as an antisymmetrical function about $\gamma = 0$. There is a maximum and a minimum value for each plot, and the peak value becomes more pronounced near to the critical point $\gamma = 0$. This indicates that the two-dimensional XY system displays a second-order QPT. Comparing the six subgraphs, we find that the absolute peak value of QD and MIN are larger than the other four measures. This means that the QD and MIN are more sensitive than the other quantum correlation measures to detect the QPT.

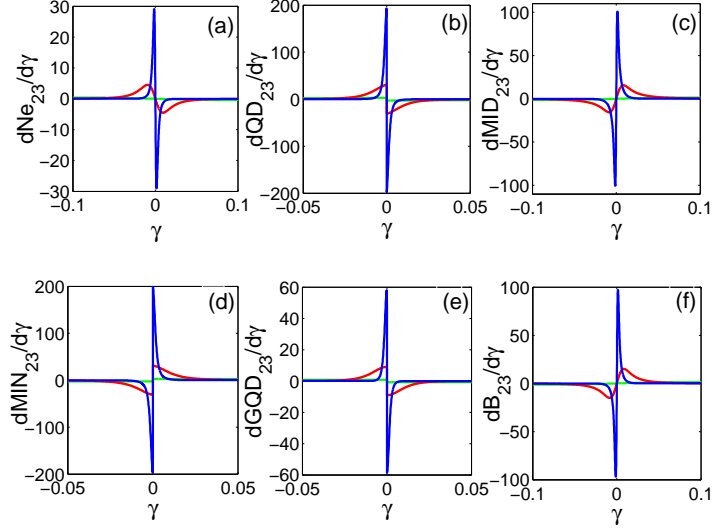


FIG. 5: (Color online) Evolution of the first derivative quantum correlation measures under QRG for state ρ_{23} . The plots color meaning are like before.

Figure 5 shows the first derivatives of the quantum correlation measures as a function of γ after tracing out block 1, 4, and 5. From the figure, it is immediately seen that the change rate and peak value of ρ_{23} is substantially quicker and larger than Fig. 4. Specifically, when *MID* and *MIN* is adopted, the absolute value of the first derivative of ρ_{23} are reaching 200. This singular behavior represents a more sensitive and more pronounced property close to the critical point $\gamma = 0$ for state ρ_{23} .

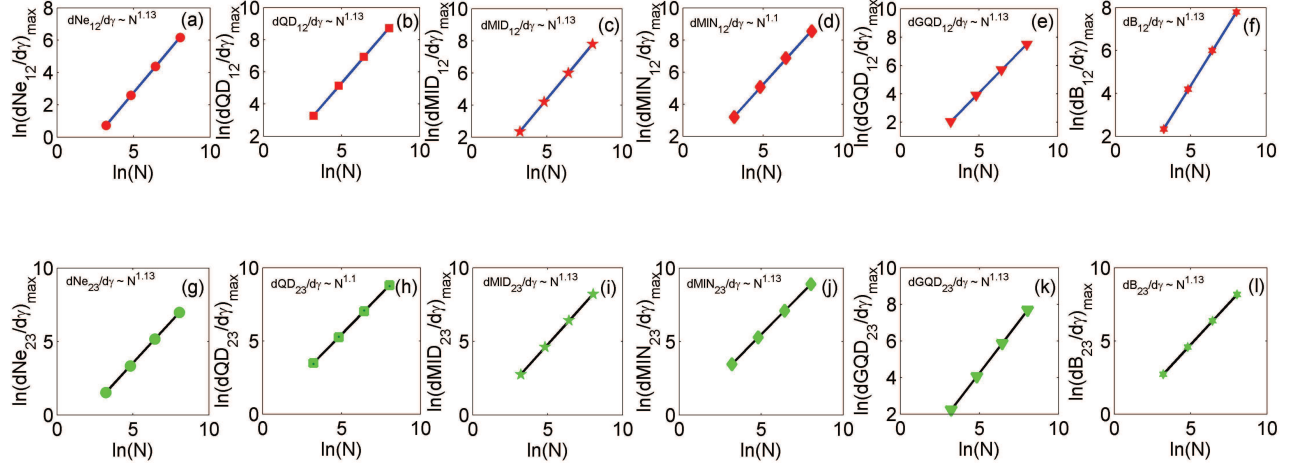


FIG. 6: (Color online) The scaling behavior of different quantum correlations in terms of system size $\ln(N)$.

We have demonstrated that the first derivative of different quantum correlation measures show the nonanalytic behavior at the critical point. A more detailed analysis shows that the maximum and minimum of the first derivative demonstrate the scaling behavior versus N . This has been depicted in Fig. 6, which displays a linear behavior of $\ln dDQCM/d\gamma$ versus $\ln N$. The scaling behavior is approximately $dDQCM/d\gamma_{max} \sim N^{1.13}$ or $dDQCM/d\gamma_{min} \sim N^{1.13}$ with exponent $\theta=1.13$. We have found that the exponent are generally identical, which means that the exponent will not change with the variation of quantum correlation measures. Since the critical exponent directly associates with the correlation length exponent [20], this result establish the relation between quantum information theory and condensed matter physics.

V. THE MONOGAMY RELATION IN TWO DIMENSIONAL XY MODEL

Monogamy relation of entanglement [4] has been a subject in the quantum information processing over the years. It is worthwhile to investigate whether the monogamy relation of entanglement and quantum correlation exist in the two-dimensional system. We also want to know whether the monogamy relation can be used to detect QPT after enough steps of the renormalization. Here we will select two typical entanglement and quantum correlation measures that is the concurrence and QD as the quantity. The concurrence[35] for a bipartite

state is $C = \text{Max}\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$, where $\lambda_k (k = 1, 2, 3, 4)$ are the square roots of the eigenvalues in descending order of the operator $R_{AB} = \rho_{AB}\tilde{\rho}_{AB}$, $\tilde{\rho}_{AB} = (\sigma_1^y \sigma_2^y) \rho_{AB}^* (\sigma_1^y \sigma_2^y)$. For a five-site block state, two kinds of the inequality in terms of concurrence is expressed by [36, 37]

$$C_{12}^2 + C_{13}^2 + C_{14}^2 + C_{15}^2 \leq C_{1|2345}^2 \quad (27)$$

$$C_{21}^2 + C_{23}^2 + C_{24}^2 + C_{25}^2 \leq C_{2|1345}^2 \quad (28)$$

where C_{ij} stands for the concurrence of the density matrix ρ with blocks other than i, j traced out, and $C_{i|jklm}$ stands for the concurrence between the subsystems ρ_i and ρ_{jklm} . The analytical expressions for C_{ij} can be computed through the above formula and $C_{i|jklm} = 2\sqrt{\rho_i}$. The difference between the two sides of inequality relation can be set as the residual entanglement that is $\delta_{1(2345)} = C_{1|2345}^2 - C_{12}^2 - C_{13}^2 - C_{14}^2 - C_{15}^2$ and $\delta_{2(1345)} = C_{2|1345}^2 - C_{21}^2 - C_{23}^2 - C_{24}^2 - C_{25}^2$.

Similarly, we can derive the monogamy inequality of QD [7]

$$QD_{12}^2 + QD_{13}^2 + QD_{14}^2 + QD_{15}^2 \leq QD_{1|2345}^2 \quad (29)$$

$$QD_{21}^2 + QD_{23}^2 + QD_{24}^2 + QD_{25}^2 \leq QD_{2|1345}^2 \quad (30)$$

here QD_{ij} have the similar meaning like for concurrence but stand for the quantum correlation, and $QD_{i|jklm} = S(\rho_i)$. We also can define the difference between the two sides of inequality relation that is $\Delta_{1(2345)} = QD_{1|2345}^2 - QD_{12}^2 - QD_{13}^2 - QD_{14}^2 - QD_{15}^2$ and $\Delta_{2(1345)} = QD_{2|1345}^2 - QD_{21}^2 - QD_{23}^2 - QD_{24}^2 - QD_{25}^2$.

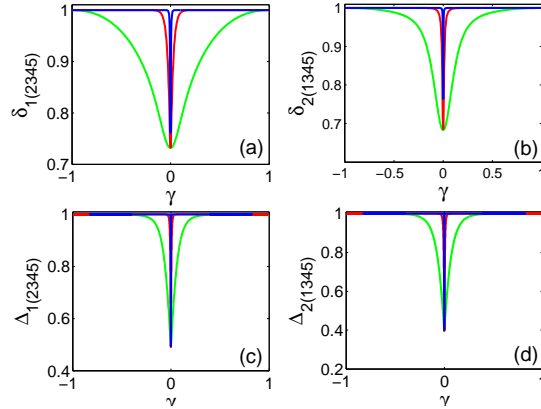


FIG. 7: (Color online) The change of δ (a) and Δ (b) of the model versus γ at different QRG steps. The plots color meaning are like before.

Numerical simulations are performed for concurrence and QD in Fig. 7, and we show that the concurrence and QD is monogamous for this two dimensional system. The curves of $\delta_{1(2345)}$, $\delta_{2(1345)}$, $\Delta_{1(2345)}$, and $\Delta_{2(1345)}$ also cross each other at $\gamma_c = 0$. This means that the residual entanglement and residual quantum correlation also can be used to indicate the QPT. The difference of the monogamy relation which is defined as monogamy score can be used as a good tool to investigate the QPT problem. Moreover, such monogamy score can characterize the genuine quantum correlation in this model[7].

VI. SUMMARY

To summarize, we have studied the renormalization of entanglement, quantum correlation, and monogamy relation properties in the two-dimensional XY model. As opposed to the one-dimensional case, the two-dimensional system size increases rapidly because we select five-site as one block. Further, the critical point and the saturated values can be reached in the lesser number of QRG iterations, this also does not hold for the one-dimensional model. The scaling behavior was investigated through determination of the quantum correlation exponent which demonstrates how the critical point is attained as the size of the model becomes large. Remarkably, we have obtained the identical critical exponent for entanglement, quantum correlation and Bell-equality. Moreover, we have studied multipartite quantum correlations with the monogamy of concurrence and monogamy of quantum discord and shown that the two quantities are monogamous in this two-dimensional model. There-

fore, our results will help us deeply understand the quantum critical problem in condensed matter physics by combing the concepts in quantum information theory.

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